Calculus A Product, Quotient and Chain Rules Review Name:

Video #1

1. Differentiate the function $y = \frac{x^2+4x+3}{\sqrt{x}}$.

$$y' = \frac{x^{1/2}(2x+4) - (x^2+4x+3)\frac{1}{2}x^{-1/2}}{x}$$

$$= \frac{\sqrt{x}(2x+4) - \frac{x^2+4x+3}{2\cancel{\sqrt{x}}}}{x} \qquad \frac{\cancel{2\sqrt{x}}}{2\sqrt{x}}$$

$$= \frac{2x(2x+4) - (x^2+4x+3)}{2x\sqrt{x}} = \frac{4x^2+8x-x^2-4x-3}{2x^{3/2}} = \boxed{\frac{3x^2+4x-3}{2x^{3/2}}}$$

$x^{2/2}x^{1/2} = x^{3/2}$

Video #2

2. Find the equation of the tangent line to $f(x) = x + \sqrt{x}$ at (1,2)

$y - y_1 = m(x - x_1)$

$m = f'(1)$

$x_1 = 1$

$y_1 = 2$

$f'(x) = 1 + \frac{1}{2}x^{-1/2} = 1 + \frac{1}{2\sqrt{x}}$

$f'(1) = 1 + \frac{1}{2} = \frac{3}{2}$

$\boxed{y - 2 = \frac{3}{2}(x-1)}$

3. Suppose that $f(2) = 3, f'(2) = 2,\ g(2) = -1$, and $g'(2) = -2$. Find the following values.

a. $\frac{d}{dx}[f(x)g(x)]$ at $x = 2$

$f(x)g'(x) + g(x)f'(x)$

$f(2)g'(2) + g(2)f'(2)$

$3(-2) + (-1)(2) = -6 - 2 = \boxed{-8}$

b. $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]$ at $x = 2$

$$\frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2} = \frac{(-1)(2) - (3)(-2)}{1} = \boxed{4}$$

c. $\frac{d}{dx}\left[\frac{g(x)}{f(x)}\right]$ at $x = 2$

$$\frac{f(x)g'(x) - g(x)f'(x)}{(f(x))^2} = \frac{3(-2) - (-1)(2)}{9} = \boxed{\frac{-4}{9}}$$

video 3

4. If f is a differentiable function, find an expression for the derivative of each of the following function.

a. $y = \boxed{x^2}\boxed{f(x)}$

$y' = x^2 f'(x) + f(x)\cdot 2x$

$= \boxed{x^2 f'(x) + 2x f(x)}$

b. $y = \frac{f(x)}{x^2}$

$$y' = \frac{x^2 f'(x) - f(x)(2x)}{x^4} = \boxed{\frac{x^2 f'(x) - 2x f(x)}{x^4}}$$

5. Find the derivative of the following functions

a. $y = (x^3 - 1)^{100}$

$y' = 100(x^3-1)^{99}(3x^2) = \boxed{300x^2(x^3-1)^{99}}$

b. $y = \cot^2(\sin\theta)$ x^2

cotx sinθ

$y' = 2\cot(\sin\theta)[-\csc^2(\sin\theta)]\cos\theta$

$= \boxed{-2\cos\theta\cot(\sin\theta)\csc^2(\sin\theta)}$

c. $f(x) = \frac{1}{\sqrt[3]{x^2+x+1}} = (x^2+x+1)^{-1/3}$

$y' = -\frac{1}{3}(x^2+x+1)^{-4/3}(2x+1)$

$= \boxed{-\frac{2x+1}{3(x^2+x+1)^{4/3}}}$

video 4

d. $g(t) = \left(\frac{t-2}{2t+1}\right)^9$

$g'(t) = 9\left(\frac{t-2}{2t+1}\right)^8\left[\frac{(2t+1)(1)-(t-2)(2)}{(2t+1)^2}\right]$

$= 9\left(\frac{t-2}{2t+1}\right)^8\left[\frac{2t+1-2t+4}{(2t+1)^2}\right] = \boxed{9\left(\frac{t-2}{2t+1}\right)^8\frac{5}{(2t+1)^2}}$

e. $y = \sin(\cos(\tan x))$

$y' = \cos(\cos(\tan x))(-\sin(\tan x))\sec^2 x$

$= \boxed{-\cos(\cos(\tan x))\sin(\tan x)\sec^2 x}$

6. State the derivative of $h(x) = f(g(x))$.

$\boxed{h'(x) = f'(g(x))\, g'(x)}$

7. A table of values for f, g, f', and g' is given below. Use the information in #6 above to evaluate the following derivatives.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	3	2	4	6
2	1	8	5	7
3	7	2	7	9

a. If $h(x) = f(g(x))$, find $h'(1)$

$h'(x) = f'(g(x))\, g'(x)$

$h'(1) = f'(g(1))\, g'(1) = 5 \cdot 6 = \boxed{30}$

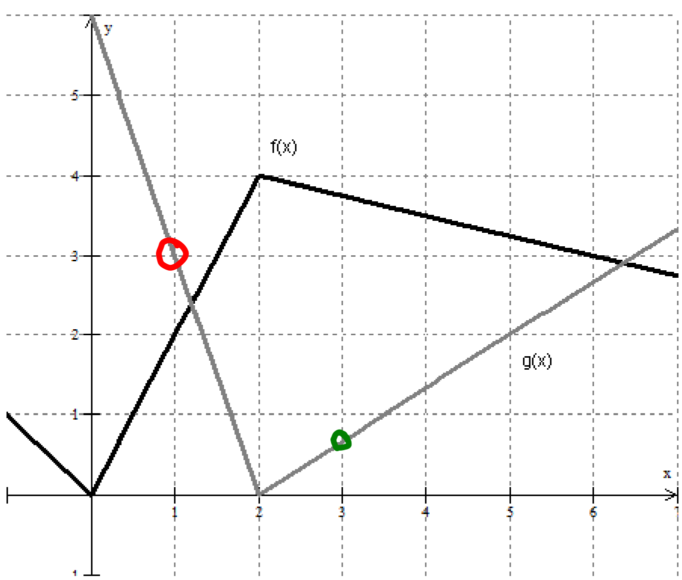
b. If $H(x) = g(f(x))$, find $H'(1)$

$H'(x) = g'(f(x))\, f'(x)$

$H'(1) = g'(f(1))\, f'(1) = 9 \cdot 4 = \boxed{36}$

video 4

8. If f and g are the functions whose graphs are shown below, let $u(x) = f(g(x))$, $v(x) = g(f(x))$, and $w(x) = g(g(x))$. Find each derivative, if it exists. If it does not exist, explain why. Remember that the derivative of a function at a point is the slope of the tangent to the curve there.



a. $u'(1)$

$f'(g(1))\, g'(1) = f'(3)\, g'(1)$

$= -\frac{1}{4}(-3) = \boxed{\frac{3}{4}}$

b. $v'(1)$

$g'(f(1))\, f'(1) = g'(2)\, f'(1)$

dne because $g'(2)$ dne

c. $w'(1)$

$g'(g(1))\, g'(1) = g'(3)\, g'(1)$

$\frac{2}{3}(-3) = \boxed{-2}$

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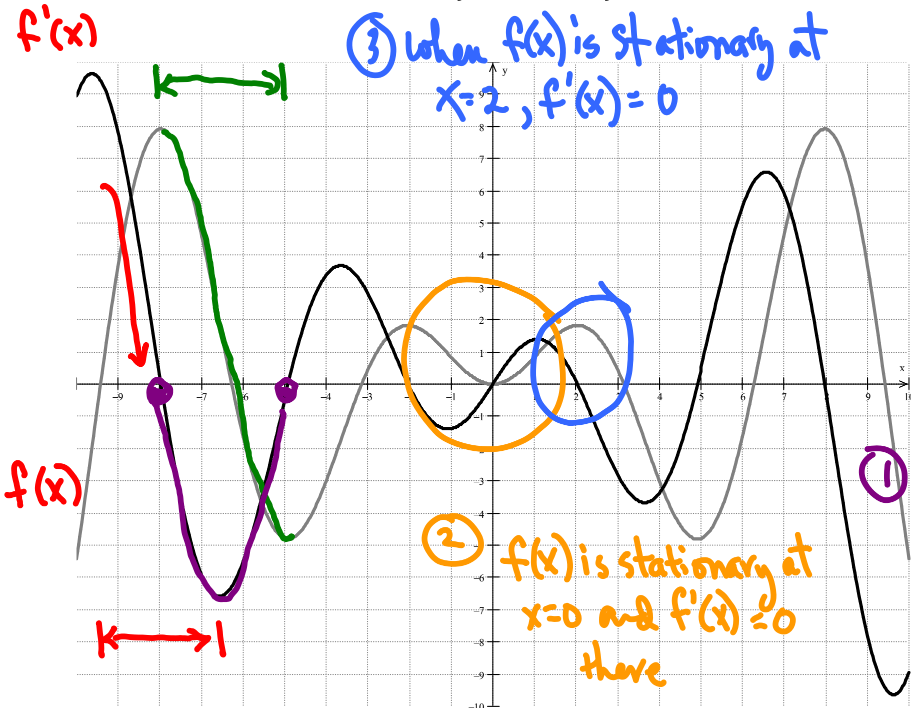
9. In the graph below, label $f(x)$ and $f'(x)$ appropriately.



when f(x) increases, f'(x) is positive
when f(x) decreases, f'(x) is negative
when f(x) is stationary, f'(x) is zero

① f(x) decreases on (-8,-5) and f'(x) is negative there as well

10. Give at least three specific points of evidence for your decision in 9 above.

a. f(x) decreases on (-8,-5) and f'(x) is negative there as well

(there are other possible answers as well)

b. f(x) is stationary at x=0 and f'(x)=0 there

(there are other possible answers as well)

c. when f(x) is stationary at x=2 f'(x)=0 there

(there are other possible answers as well)